Betting game 2 (rational beliefs = prob theory)

Def sample space X of simple events (possible outcomes)

- e.g. experiment flipping two coins X={HH, HT, TH, TT}

Let A be an event (a subset of X). A holding true = at least one of the outcomes in A happened

- e.g. "at least one heads" $\leftrightarrow$ A={HH, HT, TH}

Setup

- Assuming all unit wagers
- Writing p_A for belief of event A (your wager on A happening)

Will show that if $\{p_A\}_{A \subseteq X}$ are rational beliefs $\rightarrow$ $\{p_A\}_{A \subseteq X}$ must satisfy laws of probability theory

Betting game 2 (rational beliefs = prob theory)

- $0 \leq p_A \leq 1 \qquad \forall A \subseteq X$
  - $p > 1$: I sell you £p priced wager; You lose regardless of outcome
    - you give me £p > 1 for the wager, I need to give you at most £1
    - so $p > 1$ forms a Dutch book and is not rational
  - $p < 0$: I buy from you for £p
    - you give me £p to sell me the wager, and then you give me £1 if heads

Betting game 2 (rational beliefs = prob theory)

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- $0 \le p_A \le 1 \qquad \forall A \subseteq X$
- Unit measure $p_X = 1$ (hint: what wager would you define?)

Exercise (discuss with your neighbour)

Betting game 2 (rational beliefs = prob theory)

- $0 \leq p_A \leq 1 \qquad \forall A \subseteq X$
- Unit measure $p_X = 1$ (hint: what wager would you define?)
  - Set p to be your belief that 'at least one outcome in X holding true'
  - $p > 1$: from prev statement
  - $p < 1$: I will buy from you the wager 'at least one outcome in X holding true' at £p
    - at least one of the outcomes in X must hold, therefore you have to give me £1 and you lost £1-p.

Betting game 2 (rational beliefs = prob theory)

- $0 \leq p_A \leq 1 \qquad \forall A \subseteq X$
- Unit measure $p_X = 1$
- Two disjoint events satisfy $p_{A \cup B} = p_A + p_B$

Exercise (discuss with your neighbour)

Betting game 2 (rational beliefs = prob theory)

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- $0 \leq p_A \leq 1 \quad \forall A \subseteq X$
- Unit measure $p_X = 1$
- Two disjoint events satisfy $p_{A \cup B} = p_A + p_B$
  - set p_A your price of a promise to pay £1 if event A happens,
  - and p_B your price of a promise to pay £1 if B happens,
  - and finally $p_{A \cup B}$ price of a promise to pay £1 if either A or B happen.
  - if $p_A + p_B > p_{A \cup B}$, then
    1. I will buy wager $A \cup B$ and sell you wagers A and B;
    2. regardless of which of three outcomes happens, you lose (if A, B or none, you give me $p_A + p_B - p_{A \cup B} > 0$; note A and B disjoint)
  - opposite if price of $A \cup B$ wager too high vs other two prices

(can be extended to a countable sequence of disjoint sets)

Betting game 2 (rational beliefs = prob theory)

- $0 \leq p_A \leq 1 \qquad \forall A \subseteq X$
- Unit measure $p_X = 1$
- Two disjoint events satisfy $p_{A \cup B} = p_A + p_B$
- All laws of probability follow from these three properties of rational beliefs
- This is known as the **Bayesian** interpretation of probability theory (vs the Frequentist interpretation...)

Betting game 2 (rational beliefs = prob theory)

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